Development of the Fuzzy Traffic Assignment Model

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Abstract— **This paper defines a fuzzy traffic assignment model. First, the route choice principle is defined on 3 traffic assignment models for the maximization of possibility measure, the maximization of degree of utility, and the minimization of disutility. In the traffic assignment model for maximization of possibility measure, it is proved that the assignment result isn't affected by the maximum travel time. The traffic assignment model for maximization of degree of utility is applied to the road network in the Nagoya Metropolitan Region, and compared with the user equilibrium model, etc.**

I. INTRODUCTION

The User Equilibrium traffic assignment model, which is a fundamental model of the traffic assignment model, handles the traffic equilibrium problem according to following principle.

The Wardropian principle of equal travel times [1]:

> *Traffic on a network distributes itself in such a way that the travel costs on all routes used from any origin to any destination are equal while all unused routes have equal or greater travel costs.*

However, the prerequisites shown below are necessary for this principle to be satisfied.

- All users act in a way that always minimizes travel time.
- All users have always obtained perfect information on the available route.

In Japan, the total number of VICS(Vehicle Information and Communication System) units shipped had reached 4,488,610 by the end of March 2002 [2], and it is expected to increase in the future. However, these prerequisites seem very strong.

The Stochastic User Equilibrium traffic assignment model based on the logit formula was developed [3] to overcome these unrealistic prerequisites. This model introduces an error term into the user's utility function. This can be classified into two uncertainties depending on the route choice.

- Uncertainty in recognition of travel time
- Diversity of drivers' the route choice

For actual road traffic, it is difficult to accurately predict travel time in each route, and a driver assumes a range of travel time for each route. It is not possible to accurately express travel time by human recognition if the random utility model is used. However, it is possible to express it more accuracy if the travel time in each route is expressed by a fuzzy number. Therefore, fuzzy theory seems to be more appropriate than the random utility model. The traffic assignment model using variational inequality with fuzzy functions was developed by Liao and Wang [4], the possibilistic traffic assignment model was developed by Chang and Chen [5], and the fuzzy route choice model was developed by Henn [6]. The fuzzy shortest path algorithm by maximizing the possibility measure was developed by Ito and Ishii [7], and the traffic assignment model using this algorithm was also examined by Akiyama and Kawahara [8]. However, in this method the available route of the each OD pairs must be known, and it is difficult to apply to the real road network. Furthermore, if the method of maximizing the possibility measure is applied, only the shortest travel time is considered. Thus, this model is insufficient. In this study, the traffic assignment method for a large-scale network that models the travel time uncertainty with minimum and maximum values is developed.

II. ASSIGNMENT PRINCIPLE OF THE FUZZY TRAFFIC ASSIGNMENT MODEL

Traffic assignment models include the User Equilibrium traffic assignment model, the User Optimal traffic assignment model, and the System Optimal traffic assignment model, each having different assignment principles. It is also possible to define several kinds of assignment principle by using fuzzy theory. The following principles are defined: the Traffic Assignment Model for Maximization of Possibility Measure, the Traffic Assignment Model for Maximization of Degree of Utility, and the Traffic Assignment Model for Minimization of Disutility.

A. Traffic assignment model for maximization of possibility measure

The assignment principle of the maximization of possibility measure assignment model is defined as follows:

> *Within the available route, the maximum route of possibility of "travel time becomes less than B" is chosen.*

Although the travel time for each route is different for every run, it is possible to define it as a set of near values. Using L-R fuzzy number, the membership function in travel time in each of these routes $\mu_{\tilde{D}_{ij}}(x)$ is defined by equation (1).

$$\mu_{\tilde{D}_{ij}}(x) = \begin{cases} L\left(\dfrac{m_{ij} - x}{\alpha_{ij}}\right) & \left(x \le m_{ij}\right) \\ R\left(\dfrac{x - m_{ij}}{\beta_{ij}}\right) & \left(x \ge m_{ij}\right) \end{cases} \tag{1}$$

subject to

$\alpha_{ij}, \beta_{ij} > 0$,

L : strict decrease function $L(0)=1; L:[0,+\infty)\rightarrow[0,1]$,
R : strict decrease function $R(0)=1; R:[0,+\infty)\rightarrow[0,1]$.

This membership function is shown in Fig.1.

Next, the fuzzy goal G is set.

G: Travel time of the route is mostly under B.

In this study, B is calculated on the basis of the travel time of the shortest path between O-D. Therefore, the membership function of G is as defined in equation (2), shown in Fig.2.

$$\mu_G(x)=\begin{cases}1 & (0<x<B)\\ \dfrac{C-x}{C-B} & (B\le x\le C)\\ 0 & (x>C)\end{cases} \qquad (2)$$

The assignment route of the maximization of possibility measure model is formulated in equation (3).

$$Max.\ \prod_{\tilde{D}(p)}(G) \qquad (3)$$
$$subject\ to\quad p\in P_n$$

$\prod_{\tilde{D}(p)}(G)$ is possibility measure for fuzzy goal G.

$$\prod_{\tilde{D}(p)}(G)=\sup_x\ \min\{\mu_{\tilde{D}(p)}(x),\mu_G(x)\} \qquad (4)$$

P_i is the route set from node 1 to node i. In Fig.3, the y coordinate value of the intersection point of the membership function of the travel time and the membership function of the fuzzy goal becomes the possibility measure. The traffic is assigned the route where this possibility measure becomes a maximum.

B. Traffic assignment model for maximization of degree of utility

The assignment principle of maximization of degree of utility assignment model is defined as follows:

Within the available route, the maximum route of possibility of "satisfy" is chosen.

The assignment route of maximization of degree of utility assignment is formulated in equation (5).

$$Max.\ \frac{\int\min\{\mu_{\tilde{D}(p)}(x),\mu_G(x)\}dx}{\int\mu_{\tilde{D}(p)}(x)dx} \qquad (5)$$
$$subject\ to\quad p\in P_n$$

In Fig.4, among the membership functions for the travel time of each route, the route that has a maximum ratio of fuzzy goal achievement is the subject of assignment.

C. Traffic assignment model for minimization of disutility

The assignment principle of minimization of disutility assignment model is defined as follows:

Within the available route, the minimum route of "not satisfy" is chosen.

Then, the assignment route of the minimization of disutility assignment is formulated in equation (6).

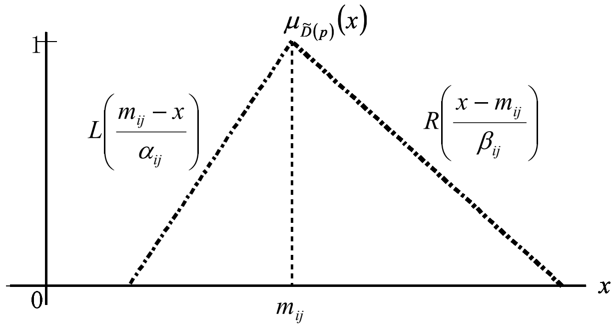


Fig.1 The Membership Function for the Travel Time

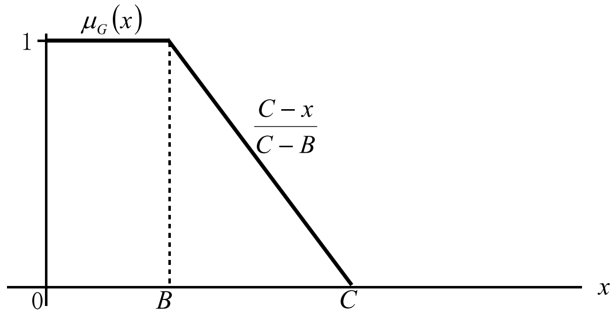


Fig.2 The Membership Function for the Fuzzy Goal

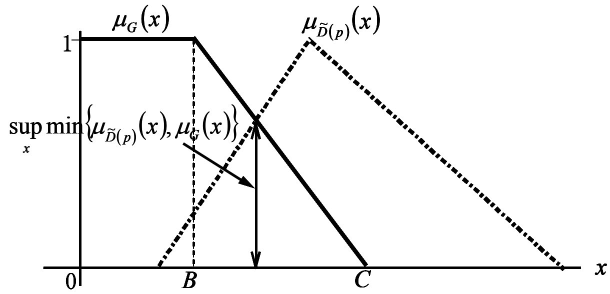


Fig.3 Maximization of Possibility Measure

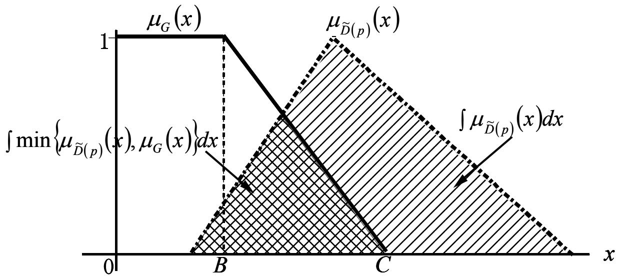


Fig.4 Maximization of Degree of Utility

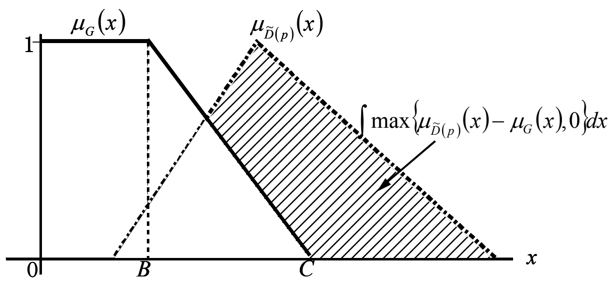


Fig.5 Minimization of Disutility

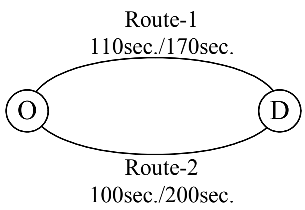
$$Min.\ \int \max\left\{\mu_{\tilde{D}(p)}(x) - \mu_G(x), 0\right\} dx \qquad (6)$$

$$subject\ to\quad p \in P_n$$

The route that minimizes the area of dissatisfaction is chosen, as shown in Fig.5.

The traffic assignment model for the maximization of possibility measure have the problem that only the left side of the membership function of travel time is considered. The traffic assignment model for minimization of disutility is an action principle where drivers have a constraint of arrival time. In contrast, the traffic assignment model for maximization of degree of utility is an action principle where drivers have no strong constraint in arrival time, and normal drivers will adopt.

For example, 1 O-D pair, the highway network of two routes shown in Fig.6 is examined. In order to show the characteristics of this method, we set the following case. The traffic capacity is 1500 vehicles for both routes. The link length is 1 km for both routes. The link performance function adopts that of multilane arterial. In Route-1, the minimum travel time and the maximum travel time are standard values calculated from the link performance function. In Route-2, it is assumed that the dispersion of travel time is longer than that of Route-1. In other words, the minimum travel time is shorter and the maximum travel time is longer than those of Route-1. The O-D traffic volume is 2000 vehicles.



minimum travel time / maximum travel time

Fig.6 Example of the Highway Network

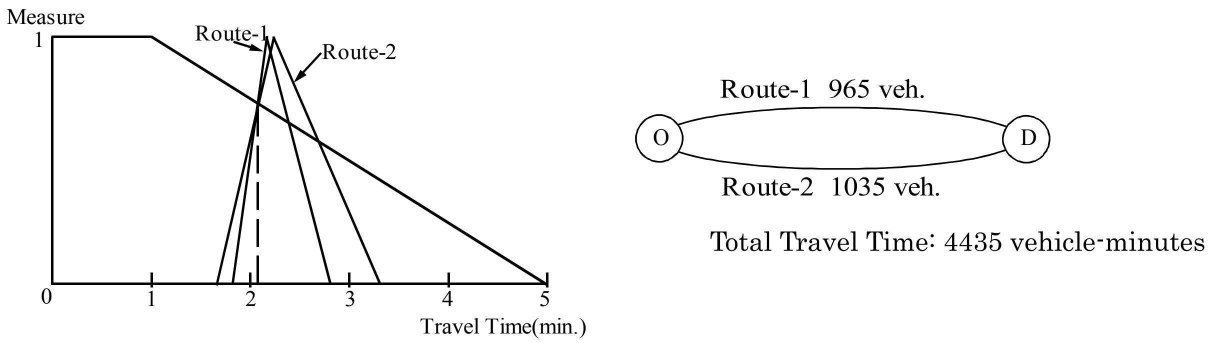


Fig.7 Traffic Assignment Model for Maximization of Possibility Measure

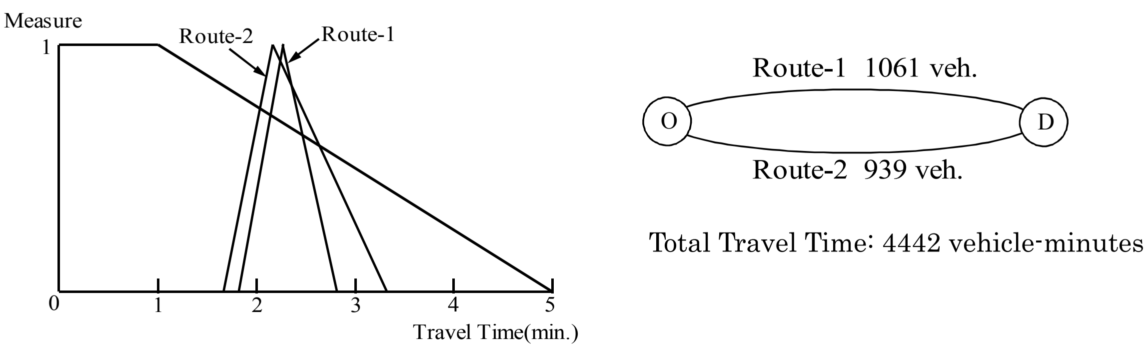


Fig.8 Traffic Assignment Model for Maximization of Degree of Utility

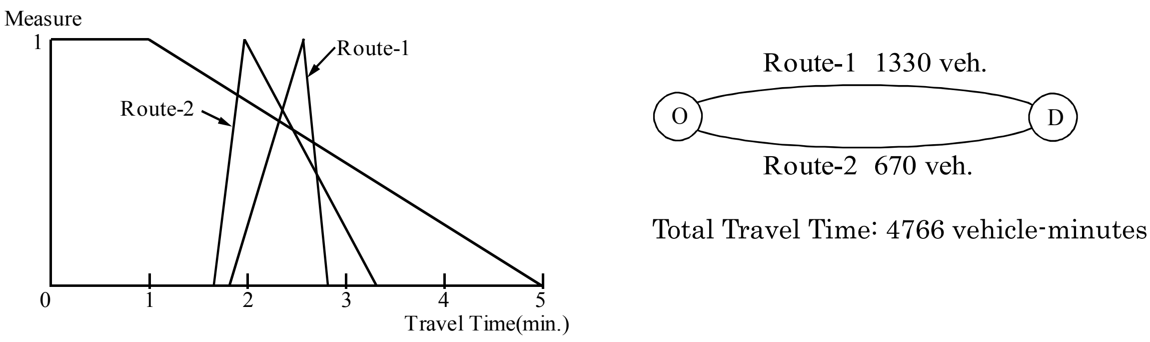


Fig.9 Traffic Assignment Model for Minimization of Disutility

The minimum and maximum travel times in each link are constant in the assignment phase, and the peak time in each link is the variable.

For the User Equilibrium traffic assignment model or the Stochastic User Equilibrium traffic assignment model, the equilibrium traffic volumes are 1000 vehicles for both routes, since conditions of traffic capacity and link length are the same. For the traffic assignment model for maximization of possibility measure, Route-2 is assigned more vehicles, since the minimum travel time of Route-2 is shorter than that of Route-1. This assignment status is shown in Fig.7. For the traffic assignment model for maximization of degree of utility, Route-1 is assigned more vehicles, since the dispersion of travel time is smaller. This assignment status is shown in Fig.8. For the traffic assignment model for minimization of disutility, Route-1 is assigned more vehicles, since the maximum travel time of Route-2 is longer, and users dislike this route.

The total travel time of the conventional equilibrium traffic assignment model is 4431 vehicle-minutes. That of the maximization of possibility measure is 4435 vehicle-minutes, that of the maximization of degree of utility is 4442 vehicle-minutes, and that of the minimization of disutility is 4766 vehicle-minutes. The total travel time of the traffic assignment model for minimization of disutility is 7% greater than that of the conventional equilibrium traffic assignment model. Since the link performance functions of Route-1 and Route-2 are the same, the total travel time for the assigned equal volume is smallest.

In this model, the peak time of the membership function is derived from the link performance function. Therefore, the calculation result may vary from the minimum and maximum travel time because of the way the minimum and maximum travel times are set. However, it is still possible to calculate it.

Next, the equilibrium traffic volume, which changes the minimum travel time of Route-2, is shown in Fig.10. The conditions of each link and OD traffic volume are the same as for the previous example, except the minimum travel time of Route-2. In the model for minimization of disutility, users dislike Route-2, as its travel time is shorter. This model is based on the principle in which the area that does not satisfy the fuzzy goal within the membership function of the travel time is minimized. Decreasing the minimum travel time means increases the dispersion of travel time. Therefore, decreasing the minimum travel time makers users dislike this route, since the area of the membership function of the travel time increases. In the traffic assignment model for maximization of possibility measure, when the minimum travel time of Route-2 equals that of Route-1 (110 seconds), the traffic volumes are 1000 vehicles for both routes. In the traffic assignment model for maximization of degree of utility, when the minimum travel time of Route-2 is 64 seconds, the traffic volumes are 1000 vehicles for both routes. This indicates that when the maximum travel time of Route-2 is longer than that of Route-1, the degree of utility will not match under the same traffic volume unless the minimum travel time of Route-2 is 46 seconds shorter. Therefore, it

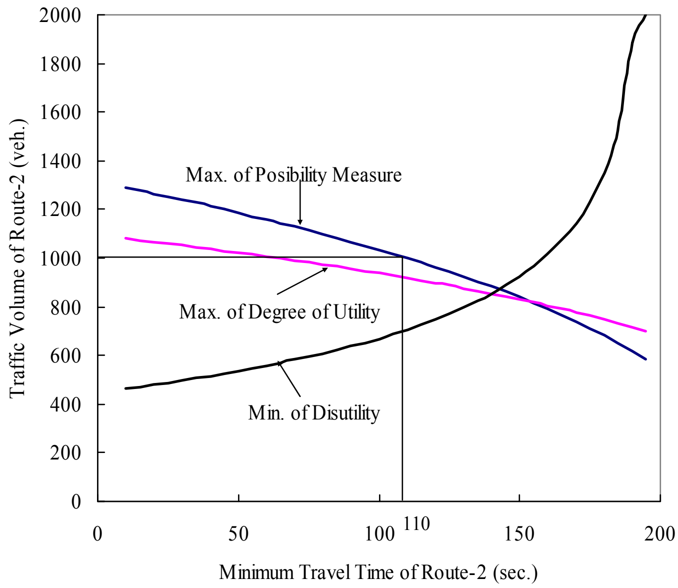


Fig.10 The Equilibrium Traffic Volume, which Changes the Minimum Travel Time of Route-2

can be said that the maximum travel time affects the degree of utility more than the minimum travel time. Since the membership function of the fuzzy goal is the decrease function, fluctuation of the maximum travel time has more effect on the area of non-satisfaction of the fuzzy goal in the membership function of travel time.

The traffic assignment model for minimization of disutility has the contradiction that the utility of the route decreases when the minimum travel time decreases. Thus, this model is illogical.

III. Examination Of Assignment Accuracy Using A Real Highway Network

A. *Outline of data*

The target area is the Nagoya urban area in Japan shown in Fig.11. There are 4303 links and 1304 nodes. The zone division adopts the basic zone of the person trip survey. There are 279 centroids. The O-D traffic volume adopts a rush hour from 7:00 to 9:00.

The minimum travel time is calculated by the link performance function of zero flow, and the maximum travel time is calculated from the traffic capacity.

B. *Assignment result*

Fig.12 compares the assignment result of the traffic assignment model for maximization of degree of utility with the real traffic volume. For this comparison, the assignment is carried out using the User Equilibrium traffic assignment model, and the Stochastic User Equilibrium traffic assignment model. Indexes such as RMS error in each assignment result are shown in Table1.

The assignment result is not good in any of the assignment models. There seems to be a problem not only in the assignment models but also in the accuracy of the O-D data.

In the Stochastic User Equilibrium traffic assignment model, parameter θ, which shows the user dispersion, is set to 1.0. Compared with the User Equilibrium traffic assignment model, goodness of fit decreased for all indexes. In this case, it is considered that goodness of fit increases, as the user dispersion is less. Similarly, goodness of fit of assignment outcome of the maximization degree of utility model is lower than that of the User Equilibrium traffic assignment model.

Next, the assignment result is examined for every road classification. In the inter-urban expressway, the correlation coefficients are very bad for every assignment model, since there are very few observation points of traffic volume. However, the other indexes are not so bad in comparison with other road classifications. In the urban expressway, the User Equilibrium traffic assignment model is better suited than the other assignment models. In the surface street, there is seldom a difference according to the assignment model.

In the traffic assignment model for maximization of degree of utility, the appropriate value of the minimum and maximum travel times are currently unknown. Although the minimum and maximum travel times are calculated by the link performance function in this study, this assumption may be not appropriate. In fact, some highways have sections that always suffer from traffic congestion and sections that always run smoothly. Minimum and maximum travel times should take these factors into account. It is necessary to set a minimum and maximum travel time that is appropriate to every road classification or every highway section in future. The developed model which can consider the minimum and maximum travel times has a possibility of flexible following the change of future traffic situation.

IV. Summary

This study shows that the range of time can be closer to human recognition if the travel time of each route is assumed as a fuzzy number and that fuzzy theory is more appropriate for the traffic assignment model than the random utility theory. Problems of previous studies are pointed out on this basis.

Next, the assignment principle was defined for 3 traffic assignment models: for maximization of possibility measure, for maximization of degree of utility, and for minimization of disutility. The formulation was done on the model for maximization of degree of utility, and the model for minimization of disutility. The characteristic of these assignment models was examined using the highway network on 2 routes. The assignment results differed depending on the difference between the assignment principles. In the traffic assignment model for maximization of possibility measure, only the minimum travel time was considered. In the traffic assignment model for maximization of degree of utility, the maximum travel time has more affect on the utility of the route than the minimum travel time. In the traffic assignment model for minimization of disutility, it tends to dislike the route when the minimum travel time decreases, because the dispersion in travel time increases.

The assignment model of the development was applied to the highway network in the Nagoya urban area in Japan, and it

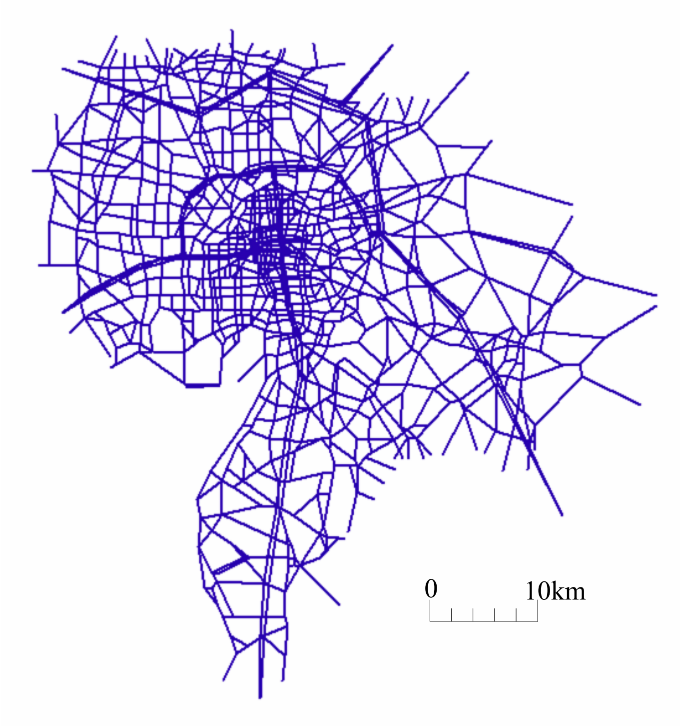


Fig.11 The Nagoya Urban Area in Japan

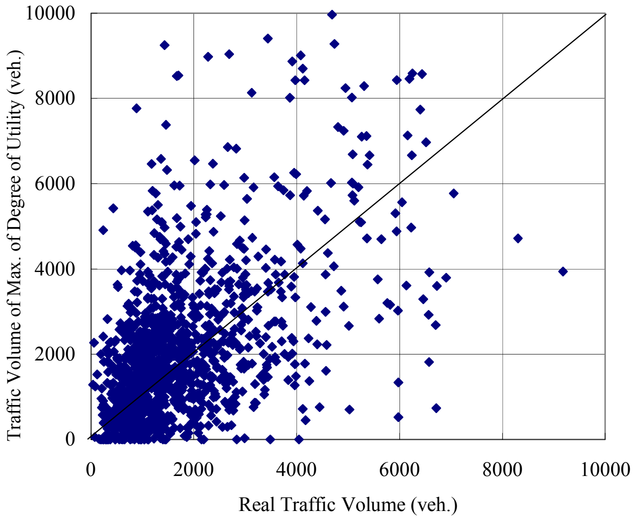


Fig.12 Comparison the Assignment Result with the Real Traffic Volume

was compared with the User Equilibrium traffic assignment model, etc. In the evaluation indexes such as the RMS error, the User Equilibrium traffic assignment model had the highest fidelity based on assignment calculation. It was proven that an appropriate minimum and maximum travel time had to be set in the traffic assignment model for maximization of degree of utility in every road classification or every highway section.

At present, research on the fuzzy shortest path problem has been spotlighted. Efficient algorithms have been developed for the fuzzy shortest path [9]. All methods must be compared and examined as future works.

Table1 Comparison of Goodness of Fit

Road Class	Index	The Model for Maximization of Degree of Utility	The User Equilibrium Model	The Stochastic User Equilibrium Model
All (1303)	e	1,178	1,042	1,210
	$Abs.RMS$	1,780	1,484	1,769
	$\%.RMS$	0.99	0.83	0.98
	$MAPE$	91.7	85.9	94.4
	r	0.59	0.57	0.23
Inter-Urban Expressway (13)	e	3,454	3,074	4,512
	$Abs.RMS$	4,208	3,811	4,751
	$\%.RMS$	0.81	0.73	0.91
	$MAPE$	72.4	63.7	85.0
	r	-0.41	-0.41	-0.24
Urban Expressway (49)	e	4,062	1,976	4,816
	$Abs.RMS$	5,134	2,407	5,067
	$\%.RMS$	1.05	0.49	1.04
	$MAPE$	112	46.4	97.8
	r	0.28	0.50	0.20
Multi-lane Arterial (778)	e	1,177	1,101	1,162
	$Abs.RMS$	1,626	1,544	1,574
	$\%.RMS$	0.85	0.80	0.82
	$MAPE$	79.6	75.8	82.7
	r	0.38	0.47	0.41
Two-lane Arterial (470)	e	815	790	822
	$Abs.RMS$	1,105	1,089	1,148
	$\%.RMS$	0.94	0.92	0.97
	$MAPE$	110	107	113
	r	0.39	0.50	0.49

$$e = \frac{1}{n}\sum_{i=1}^{n}\left|P_i - A_i\right|,\quad Abs.RMS = \sqrt{\frac{\sum_{i=1}^{n}(P_i - A_i)^2}{n}},\quad \%.RMS = \frac{Abs.RMS}{\overline{A}},$$

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{P_i - A_i}{A_i}\right|\times 100,\quad r = \frac{\sum_{i=1}^{n}\left(A_i - \overline{A}\right)\left(P_i - \overline{P}\right)}{\sqrt{\sum_{i=1}^{n}\left(A_i - \overline{A}\right)^2\sum_{i=1}^{n}\left(P_i - \overline{P}\right)^2}}$$

$A_i, \overline{A}$: Observed traffic volume and its average

$P_i, \overline{P}$: Assignment traffic volume and its average

n : Number of sample

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